TAPERED MICROSTRIP TRANSMISSION LINES

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ABSTRACT

Lossless nonuniform microstrip tapers are useful for the design of matching networks, filters, etc. In this paper these tapers have been analysed. Reflection coefficient and complex impedance results are presented for exponential, parabolic and cosine squared tapers. The paper also presents typical design data for such tapers on 0.635 mm. thick alumina substrate.

Introduction

Tapered transmission lines are of interest in the design of matching networks, filters, etc., in MIC's. Analyses of tapered lines to date have dealt with structures where the propagation constant remains constant along the length of the taper. In microstrip tapered lines the effective dielectric constant ϵ_{eff} varies along the length of the line because it is a function of the characteristic impedance. Hence, existing analyses [1-5], which use graphs or the Smith chart for impedance matching problems are inadequate for dealing with microstrip line tapers.

In this work, a modified Riccati equation is derived to take the variation of the phase constant along the tapered line into account. This is then applied to exponential, cosine squared and parabolic tapers.

Derivation of Working Equations

(a) Input Impedance

To determine the expression for input impedance at a point on the line, consider the equivalent circuit of the microstrip taper (Fig. 1) shown in Fig. 2 for the input impedance Z_{in} at a distance ℓ from the point on the line where $Z_{in} = Z_\ell = r_\ell + jx_\ell$, we have, for $\beta\ell$ sufficiently small as to approximate $\tan\beta\ell$ by $\beta\ell$

$$Z_{in} = Z(\ell)\,\frac{Z_\ell + jZ(\ell)\beta\ell}{Z(\ell) + jZ_\ell\beta\ell} \qquad (1)$$

where $Z(\ell)$ is the characteristic impedance of the line at the point ℓ.

From equation (1) we have

$$\frac{dZ_{in}}{d\ell} = \frac{Z_{in}}{Z(\ell)} \cdot \frac{dZ(\ell)}{d\ell} + \frac{f(\ell) + jg(\ell)}{b(\ell) + jc(\ell)} \qquad (2)$$

where

$$b(\ell) = Z(\ell) = x_\ell\beta\ell \qquad (3)$$

$$c(\ell) = r_\ell\beta\ell \qquad (4)$$

$$f(\ell) = r_{in}\left(\frac{dZ(\ell)}{d\ell} - x_\ell a(\ell)\right) - x_{in} r_\ell a(\ell) \qquad (5)$$

$$g(\ell) = Z(\ell)\left(\frac{dZ(\ell)}{d\ell} + Z(\ell)a(\ell)\right) + x_{in}\left(\frac{dZ(\ell)}{d\ell} - x_\ell a(\ell)\right) + r_{in} r_\ell a(\ell) \qquad (6)$$

and

$$a(\ell) = \ell\,\frac{d\beta}{d\ell} + \beta \qquad (7)$$

Equation (2) is a modified complex Riccati equation. A closed form solution to it may exist depending upon the type of the taper. Also, it can always be solved on a digital computer using numerical techniques.

(b) Reflection coefficient

The reflection coefficient Γ and the input impedance Z_{in} at any point on the line are related through the equation

$$Z_{in} = \frac{1 + \Gamma}{1 - \Gamma}\,Z(\ell) \qquad (8)$$

Using equations (2) and (6) gives

$$\frac{d\Gamma}{d\ell} = \frac{1}{2}(1 - \Gamma^2)Z(\ell)\left(\frac{dZ(\ell)}{d\ell} - Z(\ell)\frac{\left(\frac{dZ(\ell)}{d\ell} + jZ_\ell a(\ell)\right)}{b(\ell) + jc(\ell)}\right) - (1 - \Gamma^2)\frac{dZ(\ell)}{d\ell} + j(1-\Gamma)^2\,\frac{Z(\ell)\left(\beta\ell\frac{dZ(\ell)}{d\ell} + Z(\ell)a(\ell)\right)}{b(\ell) + jc(\ell)} \qquad (9)$$

Again this can be solved on a digital computer using the fourth order Runge-Kutta technique. β and $\frac{d\beta}{d\ell}$ are computed using empirical expressions [6] for the static TEM parameters of the microstrip.

Results

Using equation (2), variations of the normalized input impedance along a half wave length line, for exponential, cosine squared and parabolic tapers have been shown in Figs. 3-5. Reflection coefficient results for the tapers are shown in Fig. 6. Due to the nonuniform phase shift along the line, it can be seen that the minima of the (Γ) Vs L/λ_0 curves are not located at the multiples of $0.5L/\lambda_0$ as in the case of conventional TEM lines. A comparison of the curves shows that the exponential taper offers the best performance while the cosine squared taper offers the worst among the three kinds of tapers.

A general computer program has been developed for analyzing and producing design data for such tapers. Design curves for the width of the above-mentioned tapers have been shown in Fig. 7 for a typical substrate. The curves show that it is comparatively easy to synthesize the exponential taper.

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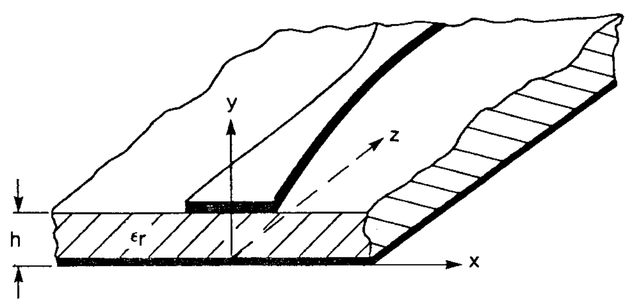

Fig. 1 Tapered Microstrip line

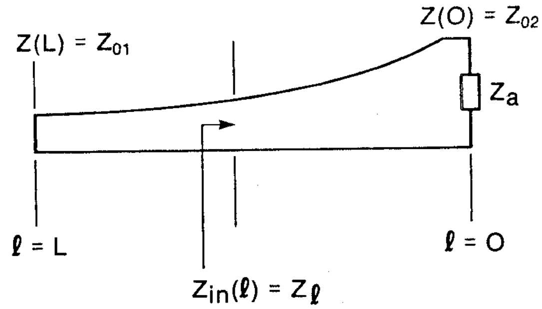

Fig. 2 Equivalent Circuit

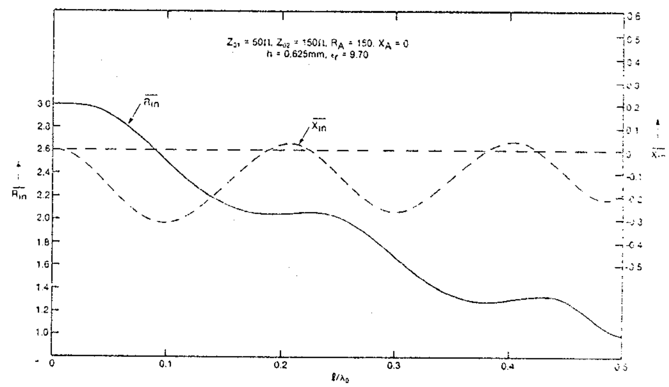

Fig. 3 Z_{in} Vs ℓ/λ_0 (Exponential Taper)

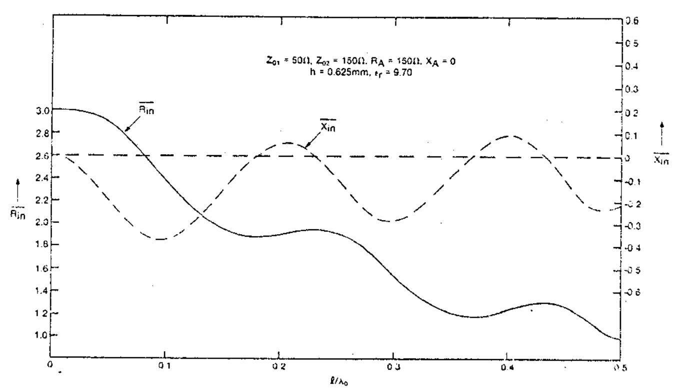

Fig. 4 Z_{in} Vs ℓ/λ_0 (Parabolic Taper)

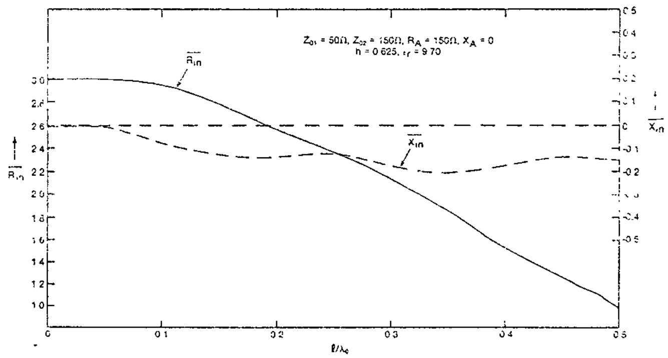

Fig. 5 Z_{in} Vs ℓ/λ_0 (Cosine-Squared Taper)

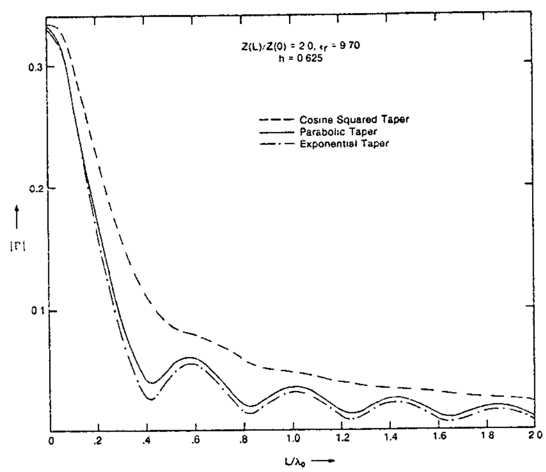

Fig. 6 Comparison of input reflection Coefficients for different lines.

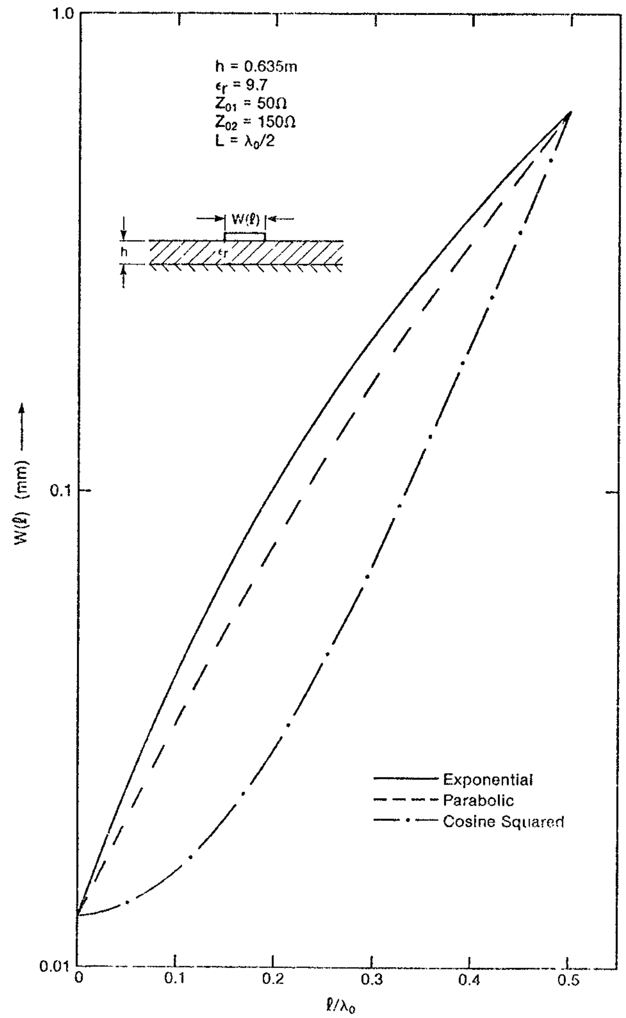

Fig. 7 Design Curves.